education

Department:
Education
REPUBLIC OF SOUTH AFRICA

T1220(E)(A8)T
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NATIONAL CERTIFICATE

MATHEMATICS N3

(16030143)

8 April (X-Paper)
09:00 – 12:00

This question paper consists of 6 pages and a 2-page formula sheet.

7. Differentiation/ Differensiasie

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

Max/Min
Maks/Min

For turning points:
Vir draaipunte: $f'(x) = 0$

8. Trigonometry/ Trigonometrie

$$\sin\theta = \frac{y}{r} = \frac{1}{cosec\theta}$$

$$\cos\theta = \frac{x}{r} = \frac{1}{sec\theta}$$

$$\tan\theta = \frac{y}{x} = \frac{1}{cot\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = cosec^2\theta$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$a^2 = b^2 + c^2 - 2bc\,\cos A$$

MATHEMATICS N3

FORMULA SHEET

Any applicable formula may also be used.

1. Factors/ Faktore

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

2. Logarithms/ Logaritmes

$$\log ab = \log a + \log b$$

$$\log \frac{a}{b} = \log a - \log b$$

$$\log_b a = \frac{\log_c a}{\log_c b}$$

$$\log a^m = m \log a$$

$$\log_b a = \frac{1}{\log_a b}$$

$$\log_a a = 1 \therefore \ln e = 1$$

$$a^{\log_a t} = t \therefore e^{\ln m} = m$$

3. Quadratic formula/ Kwadratiese formule

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

4. Parabola/ Parabool

$$y = ax^2 + bx + c$$

$$y = \frac{4ac - b^2}{4a}$$

$$x = \frac{-b}{2a}$$

5. Circle/ Sirkel

$$x^2 + y^2 = r^2$$

$$D = \frac{x^2}{4h} + h$$

$$x = \sqrt{4Dh - 4h^2}$$

6. Straight line/ Reguitlyn

$$y - y_1 = m(x - x_1)$$

Perpendicular:
Loodreg: $m_1 \cdot m_2 = -1$

Parallel lines:
Ewewydige lyne: $m_1 = m_2$

Distance:
Afstand: $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint:
Middelpunt: $P = \left(\dfrac{x_1 + x_2}{2} ; \dfrac{y_1 + y_2}{2} \right)$

Angle of inclination:
Hellingshoek: $\theta = tan^{-1} m$

QUESTION 5

5.1 Given the equation: $y = asinbx$

    5.1.1   What does a represent? (1)

    5.1.2   What does b represent? (1)

5.2 Calculate the values of a that will satisfy the following equation for $0^0 \le a \le 360^0$ if sec $a = 2,5$ (4

5.3 Make use of basic trigonometric identities to prove that:

$$\frac{1}{1+sin\beta} + \frac{1}{1-sin\beta} = 2sec^2\beta$$

(4

5.4 Simplify the following:

$$\frac{tan(180^0 - \theta)cos(360^\circ - \theta) + sin(180^\circ + \theta)}{tan(360^\circ - \theta)}$$

(4

5.5 Calculate the exact value without using the calculator:

$$\sqrt{cosec^2 150^\circ + sec^2 315^\circ - cot^2 150^\circ}$$

(3

5.6 Sketch the graphs of the following trigonometric equations each on its system of axes for: $0^\circ \le x \le 180^\circ$
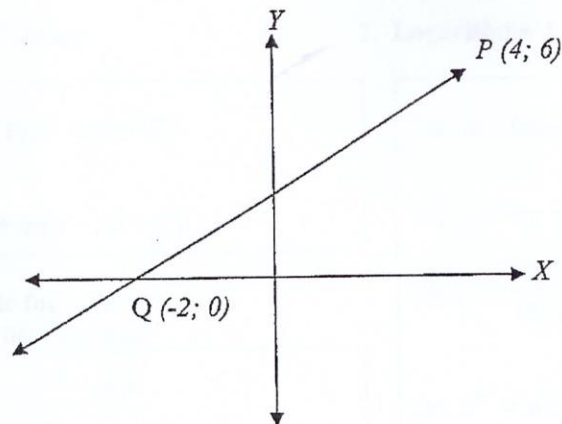
$g(x) = 3\,sin2x$

$f(x) = 2cos4x$

ALL values at the points of intersection with the system of axes and co-ordinates of the turning points must be shown.

(5

[22

TOTAL: 10

3.4 In the following diagram, points $P(4;6)$ and $Q(-2;0)$ are on a straight line in a Cartesian plane.

Calculate the following:

3.4.1 The co-ordinates of the midpoint of line PQ (2

3.4.2 The length of line PQ in simple surd form (2

3.4.3 The gradient of line PQ (2

3.4.4 The equation of line PQ in general form (3

[2

QUESTION 4

4.1 Determine the following:

$$\lim_{x \to 2} \frac{x^2 - 4x + 4}{x - 2}$$

4.2 Differentiate from first principles:

$$f(x) = 3x^2 + 2$$

4.3 Determine $\frac{dy}{dx}$ by using the rules of differentiation:

$$y = \sqrt{x} - \frac{2}{3x^3}$$ (Leave the answer with positive exponents and in surd form)

4.4 Determine the co-ordinates of the turning point of the following parabola by using differentiation:
$$y = x^2 - 2x - 3$$

2.2 Make R_s the subject of the formula:

$$V = \frac{R}{R_s + r} \times V_{dc}$$

(2

2.3 As $f = p\sqrt{1 + \frac{CL}{k^2}}$, determine k if $f = 5{,}188; p = 5{,}12; C = 0{,}003$ and $L = 23$

(3

2.4 Make I the subject of the formula by completing the square:

$$I^2 R + VI - P = 0$$

(3

2.5 The square of a natural number is four more than two times another natural number. The second number is two less than three times the first number. Determine the TWO natural numbers algebraically.

(5

[23

QUESTION 3

3.1 State whether the lines represented by the following equations are parallel or perpendicular to each other:

    3.1.1 $y = 2x - 4$ and $4y = -2x + 4$

    3.1.2 $2y = 3x + 2$ and $4y = 6x$

3.2 Given the equation of a parabola: $y = -x^2 + 4x - 4$

    3.2.1   Determine the y-intercept

    3.2.2   Calculate the x-intercepts

    3.2.3   Calculate the co-ordinates of the turning point

    3.2.4   Sketch the graph represented by the equation $y = -x^2 + 4x - 4$

3.3 Sketch the graph of $x^2 + y^2 = 4$. Use a scale of 2 $cm = 1$ unit. Show ALL values at the points of intersection with system of axes. Name the type of graph.

QUESTION 1

1.1   Fully factorise the following in prime factors:

    1.1.1 $4x^2 - (x+1)^2$ (1)

    1.1.2 $2x^2 + 5x + 3$ (2)

    1.1.3 $2x^3 + 5x + 8x^2 + 20$ (2)

1.2   Simplify the following without using the calculator:

    1.2.1 $2\log_3 2 + \log_3 10 - \log_3 3 - \log_3 40$ (2)

    1.2.2 $\dfrac{\sqrt{24} - 2\sqrt{6} + \sqrt{54}}{\sqrt{96} - \sqrt{6}}$ (3)

    1.2.3 $\dfrac{3 \times 2^x - 4 \times 2^{x-2}}{2^x - 2^{x-1}}$ (3)

1.3   Prove that $x-2$ is a factor of the following function by making use of the factor theorem:

$$f(x) = x^3 + 2x^2 - 5x - 6$$ (3)

1.4   Simplify the following fractions:

    1.4.1 $\dfrac{x^2 + x - 2}{3x + 6} \div \dfrac{x+2}{x^2 + 3x + 2}$ (3)

    1.4.2 $\dfrac{14}{x-3} + \dfrac{5}{2+x} - \dfrac{9x-5}{x^2 - x - 6}$ (4)

                                                           [23]

QUESTION 2

2.1   Solve for x:

    2.1.1 $\sqrt{-3x - 5} - 5 = x$ (3)

    2.1.2 $\log_x 25 + \log_2 8 - \log_5 5 = 4$ (3)

    2.1.3 $2^{x+2} + \dfrac{2^x}{4} + 2^x = 42$ (4)

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MATHEMATICS N3
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1.   Answer ALL the questions.

2.   Read ALL the questions carefully.

3.   Number the answers correctly according to the numbering system used in this question paper.

4.   Show ALL the calculations and intermediary steps. Simplify where possible.

5.   Questions may be answered in any sequence, but subsections of questions may NOT be separated.

6.   ALL the answers must be approximated accurately to THREE decimal places.

7.   1 mark = 1%

8.   NOTE: Diagrams are not drawn to scale.

9.   The formula sheets (attached) are not necessarily complete. Any other applicable formula may be used.

10.   Write neatly and legibly.